## Classification Trees

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- Decision Tree
- Project
- Use of data
- Don't use kaggle datasets
- Talk to us about your data in class and at consults
- Practical exam
- Next Wednesday from 12pm Wednesday, closing 12pm Thursday


## What is a decision tree?

Tree based models consist of one or more of nested ifthen statements for the predictors that partition the data. Within these partitions, a model is used to predict the outcome.


Source: Egor Dezhic

Regression Tree


Regression Tree



## Regression tree

-What if we want to predict something being in a particular group? Say,predicting whether someone passes a course based on two exam scores:

- Moving from continuous to categorical response.

- Regression trees give the predicted response for an observation by using the mean response of the observations that belong to the same terminal node:


## Classification

A classification tree predicts each observation belonging to the most commonly occurring class of observations.
However, when we interpret a classification tree, we are often interested not only in the class prediction (what is most common), but also the proportion of correct classifications.

## Building a classification tree

- Similar approach to building a classification tree as for regression trees
- We use this "recursive binary splitting" approach
- But we don't use the residual sums of squares

$$
S S_{T}=\sum\left(y_{i}-\bar{y}\right)^{2}
$$

Since we now have a category, we need some way to describe that. We need something else!

## Classification tree

- We can use the "classification error".
- Where we count up the number of mis-classified things, and choose the split that has the lowest number of mis-classified things.
- We can represent this in an equation as the fraction of observations in a region which don't belong to the most common class.
\$\$ E = 1 - \text\{max\}\{k\}(\hat\{p\}\{mk\}) \$\$
Here, $\hat{p}_{m k}$ refers to the proportion of observations in the $m$ th region, from the $k$ th class.


## Understanding classification

Another way to think about this is to understand when E is zero, and when $E$ is large
$E=1-\max _{k}\left(\hat{p}_{m k}\right)$
E is zero when $\max _{k}\left(\hat{p}_{m k}\right)$ is 1 , which is 1 when observations are the same class:

## Classification trees

- A classification tree is used to predict a categorical response and regression tree is used to predict a quantitative response
- Use a recursive binary splitting to grow a classification tree. That is, sequentially break the data into two subsets, typically using a single variable each time.
- The predicted value for a new observation, $x_{0}$, will be the most commonly occurring class of observations in the sub-region in which $x_{0}$ falls


## Predicting pass or fail ?

Consider the dataset Exam where two exam scores are given for each student, and a class Label represents whether they passed or failed the course.

| \#\# | Exam1 | Exam2 | Label |
| ---: | ---: | ---: | ---: |
| \#\# | 1 | 34.62366 | 78.02469 |
| \#\# | 2 | 30.28671 | 43.89500 |
| \#\# 3 | 35.84741 | 72.90220 | 0 |
| \#\# 4 | 60.18260 | 86.30855 | 1 |

## Your turn:

## Open "10b-exercise-intro.Rmd" and let's decide a point to split the

 data.
## Calculate the number of misclassifications

Along all splits for Exam1 classifying according to the majority class for the left and right splits


Red dots are "fails", blue dots are "passes", and crosses indicate misclassifications. Source: John Ormerod, U.Syd

## Calculate the number of misclassifications

Along all splits for Exam2 classifying according to the majority class for the top and bottom splits



Red dots are "fails", blue dots are "passes", and crosses indicate misclassifications. Source: John Ormerod, U.Syd

## Combining the results from Exam1 and Exam2 splits

- The minimum number of misclassifications from using all possible splits of Exam1 was 19 when the value of Exam1 was 56.7
- The minimum number of misclassifications from using all possible splits of Exam2 was 23 when the value of Exam2 was 52.5

So we split on the best of these, i.e., split the data on Exam1 at 56.7.

## Split criteria - purity/impurity metrics

It turns out that classification error is not sufficiently sensitive for tree-growing.
In practice two other measures are preferable, as they are more sensitive:

- The Gini Index and
- Information Entropy.

They are both quite similar numerically.
Small values mean that a node contains mostly observations of a single class, referred to as node purity.

## Example - predicting heart disease

## $Y$ : presence of heart disease (Yes/No)

$X$ : heart and lung function measurements

| \#\# [1] "Age" | "Sex" | "ChestPain" "RestBP" | "Chol" | "Fbs" |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| \#\# [7] "RestECG" | "MaxHR" | "ExAng" | "Oldpeak" | "Slope" | "Ca" |
| \#\# [13] "Thal" | "AHD" |  |  |  |  |



## Deeper trees

## Trees can be built deeper by:

- decreasing the value of the complexity parameter $c p$, which sets the difference between impurity values required to continue splitting.
- reducing the minsplit and minbucket parameters, which control the number of observations below splits are forbidden.



## Tabulate true vs predicted to make a confusion table.



- Accuracy: $(a+d) /(a+b+c+d)$
- Error: $(b+c) /(a+b+c+d)$
- Sensitivity: $a /(a+c)$ (true positive, recall)
- Specificity: $d /(b+d)$ (true negative)
- Balanced accuracy: (sensitivity+specificity)/2

Confusion and error

| \#\# | Reference |  |  |  |
| :--- | :---: | ---: | ---: | :---: |
| \#\# | Prediction | No | Yes |  |
| \#\# | No | 75 | 5 |  |
| \#\# | Yes | 11 | 58 |  |
| \#\# | Accuracy |  |  |  |
| \#\# | 0.8926174 |  |  |  |

## Example - Crabs

## Physical measurements on WA crabs, males and females.

Data source: Campbell, N. A. \& Mahon, R. J. (1974)


Example - Crabs


## Comparing models

Classification tree


Linear discriminant classifier


## Strengths and Weaknesses

Strengths:

- The decision rules provided by trees are very easy to explain, and follow. A simple classification model.
- Trees can handle a mix of predictor types, categorical and quantitative.
- Trees efficiently operate when there are missing values in the predictors.
Weaknesses:
- Algorithm is greedy, a better final solution might be obtained by taking a second best split earlier.
- When separation is in linear combinations of variables trees struggle to provide a good classification


## Made by a human with a computer

- Slides inspired by https://iml.numbat.space, https://github.com/numbats/iml.
- Created using R Markdown with flair by xaringan, and kunoichi (female ninja) style.
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